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Exact propagators for Dirac particle with anomalous magnetic moment in a plane wave field

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Abstract. The propagators related to a Dirac particle with anomalous magnetic moment in the presence of a plane wave field are exactly and analytically calculated via the path integral approach in two representations: a global and a local one. It is shown that due to two identities, one bosonic and another fermionic, the path integral calculations are evaluated essentially on classical paths projected along the wave propagation k. Special cases are considered as well.

1 Introduction

In spite of the achievement that the path integral has brought about in various domains of physics by trying to bring back the quantum description of the physical systems to its classical analogue, it appeared, however, unsatisfactory in use in the case of the description of the spin. This apparent inconvenience is due to the fact that the spin is a purely discrete physical entity which does not have a classical analogue, whereas the path integral is elaborated basically by means of classical images such as trajectories. In attempting to solve this difficulty, there have been essentially two categories of path integral formulation. The first one, called the bosonic model, proposes to use commuting variables to describe the spin dynamics and the second one, known as fermionic, uses Grassmann (anticommuting) variables. The latter model was proposed by Berezin and Marinov who have presented the Dirac propagator by means of a Grassmannian path integral in the form of exp(i action) [1]. This model is a renewal of the model due to Fradkin, who was the first to present an action for the Dirac particle based on Grassmann variables. In this last decade, Fradkin and Gitman have returned to this model by establishing a rigorous formulation of this path integral which is in possession of various properties, such as gauge invariance, reparametrization invariance and having a supersymmetric form [2,3]. This possibility in using the Grassmann variables at the classical level (when $\hbar \to 0$) has appeared to be very interesting because of its direct connection with the theory of superstrings and the interaction between matter and supergravity. The corresponding physical system is known as a pseudoclassical one and its pseudoclassical mechanics has been fully studied [4–7]. This ingenious attempt gave new breath to the research of the analytical and exact expressions of the relativistic spinning propagators in the presence of external fields. In this order many problems have been solved. For example, we can cite the interaction with a constant electromagnetic field [8], the case of a plane wave field [9–11] and the combination of the two configurations of fields [12]. In the case of an intense interaction field, it is necessary to take account of some effects such as the anomalous part of the magnetic moment. For this case the formulation was also elaborated [13] and the interaction with the constant field was treated via the perturbation method [14].

In what follows, we are interested in the calculations of this effect in the case of a plane wave interaction with the purpose of exploiting the constraints introduced in [9–11] which allow us to deal with the problem of the interaction of a plane wave with a remarkable simplicity. Thus, we want to add to the list of the anomaly problems one case which is exactly soluble. For this, it is first necessary to briefly recall the path integral construction. The propagator of the Dirac particle with an anomalous magnetic moment in an external electromagnetic field is the causal Green's function $S^{\mathrm{c}}(x_b, x_a)$ of the Dirac–Pauli equation

$$\left(\gamma \cdot \pi_b - m - \frac{\mu}{2}\sigma \cdot F_b\right) S^{\mathrm{c}}(x_b, x_a) = -\delta^4(x_b - x_a), \quad (1)$$

where $\pi_\mu = (\mathrm{i}\partial_\mu - gA_\mu)$, g is the electronic charge, μ describes the additional spin magnetic moment, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\sigma^{\mu\nu} = (\mathrm{i}/2)\,[\gamma^\mu, \gamma^\nu]_-$, $[\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu}$, $\eta^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$ and $\mu, \nu = \overline{0, 3}$.

The scalar product, denoted by a dot, means that $a \cdot b = a_\mu b^\mu$, $\sigma \cdot F = \sigma_{\mu\nu} \cdot F^{\mu\nu}$.

Multiplying by γ^5 on both sides of (1), we get

$$\left(\widetilde{\gamma} \cdot \pi_b - m\gamma^5 - \frac{\mu}{2}\gamma^5 \widetilde{\sigma} \cdot F_b\right) \widetilde{S}(x_b, x_a) = \delta^4(x_b - x_a), \quad (2)$$

where $\widetilde{S} = S^{\mathrm{c}}\gamma^5$, $\widetilde{\gamma}^\mu = \gamma^5\gamma^\mu$, $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \widetilde{\gamma}^5$ and $(\gamma^5)^2 = -1$. The matrices $\widetilde{\gamma}^\mu$ have the same commutation relations as the initial ones γ^μ, $[\gamma^n, \gamma^m]_+ = 2\eta^{nm}$,

$n, m = \overline{0,3}, 5$, $\eta^{nm} = \mathrm{diag}\,(1, -1, -1, -1, -1)$ and $F_{b,a} = F\,(x_{b,a})$.

The presence of the matrix γ^5 in this formulation is for solving the problems of homogenization which is, in fact, not important when the anomalous magnetic moment is absent as has been shown recently by elaborating the two representations known as the local and global representations [15].

Formally, $\widetilde{S}(x_b, x_a)$ is the matrix element in the coordinate space of the inverse Dirac–Pauli operator, namely

$$\widetilde{S}(x_b, x_a) = \langle x_b \mid \widetilde{S} \mid x_a \rangle, \tag{3}$$

with

$$\widetilde{S} = \left(\widetilde{\gamma}^\mu \pi_\mu - m\gamma^5 - \frac{\mu}{2}\gamma^5 \widetilde{\sigma}_{\mu\nu} F^{\mu\nu}\right)^{-1} = O^{-1}. \tag{4}$$

Using the Schwinger trick, we represent $\widetilde{S}$ by the following integral expressions:

$$\begin{aligned}\widetilde{S} = \widetilde{S}_{\mathrm{l}} = S_{\mathrm{l}}\gamma^5 &= \int_0^\infty \mathrm{d}\lambda \int \exp\left[\mathrm{i}\lambda\left(O^2 + \mathrm{i}\varepsilon\right) + \chi O\right] \mathrm{d}\chi, \\ = \widetilde{S}_{\mathrm{g}} = S_{\mathrm{g}}\gamma^5 &= O G_{\mathrm{g}} \\ &= O\left(-\mathrm{i}\int_0^\infty \mathrm{d}\lambda \exp\left[\mathrm{i}\lambda\left(O^2 + \mathrm{i}\varepsilon\right)\right]\right),\end{aligned} \tag{5}$$

where $\widetilde{S}_{\mathrm{l}}$ is called the local representation and $\widetilde{S}_{\mathrm{g}}$ the global one. It is clear that G_{g} verifies the following quadratic Dirac–Pauli equation:

$$O_b^2 G_{\mathrm{g}}(x_b, x_a) = \delta^4(x_b - x_a), \tag{6}$$

with

$$\begin{aligned}O^2 = \pi^2 - m^2 &- \mathrm{i}\left(m\mu + \frac{g}{2}\right) F_{\alpha\beta}\gamma^\alpha\gamma^\beta \\ &- \mathrm{i}\mu\gamma^5\gamma^\alpha \left[F_{\alpha\beta}, \pi^\beta\right]_+ + \frac{\mu^2}{4}(F_{\alpha\beta}\gamma^\alpha\gamma^\beta)^2,\end{aligned}$$

and $\widetilde{S}_{\mathrm{g}}(x_b, x_a)$ can be obtained starting from $G_{\mathrm{g}}(x_b, x_a)$:

$$\widetilde{S}_{\mathrm{g}}(x_b, x_a) = \left(\widetilde{\gamma}\cdot\pi_b - m\gamma^5 - \frac{\mu}{2}\gamma^5\widetilde{\sigma}\cdot F_b\right) G_{\mathrm{g}}(x_b, x_a). \tag{7}$$

Doing so, we formulate the anomalous magnetic moment problem in the two projections, the local and the global one. In the case of the local projection the operator projection O is replaced by a path integral using a fermionic proper time χ [13] while for the global one this operator shall act at the end of the evolution with the aim to eliminate the superfluous states corresponding to the square of the Dirac operator and the latter is written in the path integral representation following [15]. In the case of the so-called global representation, $\widetilde{S}_{\mathrm{g}}$, which is a mixed representation of path integral and projection operator, the construction of the propagator G_{g} in the case of the anomaly was not shown. To do this, it is preferable to follow step by step the path integral formulation for $\widetilde{S}_{\mathrm{l}}$. It is remarkable to see that the calculations of the global description G_{g} are similar to those of the local one $\widetilde{S}_{\mathrm{l}}$. Consequently, it is convenient to unify these two Green's functions into a λ-modified one $\widetilde{S}_\lambda$ by taking into account the λ-modified measure of χ which ensures that

$$\widetilde{S}_\lambda = \begin{cases} \widetilde{S}_{\mathrm{l}}, & \lambda = 1, \\ G_{\mathrm{g}}, & \lambda = 0. \end{cases} \tag{8}$$

In fact, knowing that $\int \mathrm{d}\chi = 0$ and $\int \mathrm{d}\chi\chi = 1$, we easily write for the Green's function $\widetilde{S}_\lambda$ the following result [13, 15]:

$$\begin{aligned}\widetilde{S}_\lambda = &\left(\frac{-\mathrm{i}}{2}\right)^{1-\lambda} \exp\left(\mathrm{i}\widetilde{\gamma}^n \frac{\partial_{\mathrm{l}}}{\partial\theta^n}\right) \int_0^\infty \mathrm{d}e_0 \int \mathrm{d}\chi_0 \\ &\times \int Dx De D\pi D\chi D\nu \mathcal{D}\Psi \mathbb{M}(e) \chi^{1-\lambda} \\ &\times \exp\left\{\mathrm{i}\int_0^1 \left[\frac{-\dot{x}^2}{2e} - \frac{e}{2}M^2\right.\right. \\ &- \dot{x}^\mu \left(gA_\mu(x) + 4\mathrm{i}\mu\Psi^5 F_{\mu\nu}(x)\Psi^\nu\right) \\ &+ \mathrm{i}egF_{\mu\nu}(x)\Psi^\mu\Psi^\nu + \mathrm{i}\lambda\left(\frac{\dot{x}_\mu \Psi^\mu}{e} - M^*\Psi^5\right)\chi \\ &\left.\left.- \mathrm{i}\Psi_n \dot{\Psi}^n + \pi\dot{e} + \nu\dot{\chi}\right]\mathrm{d}\tau + \Psi_n(1)\Psi^n(0)\right\}\Bigg|_{\theta=0},\end{aligned} \tag{9}$$

where x, p, λ, π and $\chi, \nu, \Psi^n, \theta^n$ are even and odd variables respectively and satisfy the following boundary conditions:

$$\begin{gathered} x(0) = x_a, \quad x(1) = x_b, \quad \lambda(0) = \lambda_0, \quad \chi(0) = \chi_0, \\ \Psi^n(0) + \Psi^n(1) = \theta^n, \quad M^* = m + 2\mathrm{i}\mu F_{\mu\nu}(x)\Psi^\mu\Psi^\nu, \end{gathered}$$

and the measures $\mathbb{M}(e)$ and $\mathcal{D}\Psi$ are defined by

$$\begin{aligned}\mathbb{M}(e) &= \int Dp \exp\left\{\frac{\mathrm{i}}{2}\int_0^1 ep^2 \mathrm{d}\tau\right\}, \\ \mathcal{D}\Psi &= D\Psi\left[\int_{\Psi(1)+\Psi(0)=\theta} D\Psi \exp\left(\int_0^1 \Psi_n \dot{\Psi}^n \,\mathrm{d}\tau\right)\right]^{-1}.\end{aligned}$$

The aim of this paper is to calculate this Green function $\widetilde{S}_\lambda$ in the case of a plane wave field. The plane wave field A^μ is characterized by the following properties:

$$\varphi = k\cdot x \quad \text{with } k^2 = 0 \quad \text{and } k\cdot A = k\cdot\frac{\mathrm{d}A}{\mathrm{d}\varphi} = 0. \tag{10}$$

As we are going to show, thanks to these simple properties of the plane wave field, the solution may be analytically obtained.

2 Exact Green's function calculations

Before starting the calculations, let us first expose our strategy in treating this problem. Taking advantage of the fact that the plane wave field is a function of the variable

$k\cdot x$ and that the spin interaction depends on the $k\cdot\Psi$ variable, we introduce two identities which consider these two variables as independent respectively from x and Ψ. This allows us to decouple respectively the free quadridimensional exterior motion x and the free quadridimensional interior motion Ψ from the corresponding interaction terms and consequently to reduce these two quadridimensional motions to unidimensional ones which are intimately related to the classical motion.

As usually done, we firstly integrate over π and ν to fix respectively e to e_0 and χ to χ_0, and next, as the plane wave field is only a function of the product $k\cdot x$, we introduce a new variable $\varphi = k\cdot x$ as independent from the quadriposition x via a delta functional identity [16–18],

$$\int \mathrm{d}\varphi_a \mathrm{d}\varphi_b \delta(\varphi_a - k\cdot x_a)\int D\varphi D p_\varphi \times \exp[\mathrm{i}\int_0^1 p_\varphi(\dot{\varphi} - k\cdot \dot{x})\mathrm{d}\tau] = 1, \tag{11}$$

This change permits us to separate the free action term $-\dot{x}^2/(2e_0)$ from the one of the interaction. Now, we linearize this quadratic term $-\dot{x}^2/(2e_0)$ and then integrate over the x variables. This makes appear the Dirac functional $\delta\left(\dot{p}\right)$, showing that the momentum is a constant of motion $p=$ const. Accordingly, the propagator is reduced to

$$\begin{aligned}\widetilde{S}_\lambda = &\left(\frac{-\mathrm{i}}{2}\right)^{1-\lambda}\exp\left(\mathrm{i}\widetilde{\gamma}^n\frac{\partial_1}{\partial\theta^n}\right)\int\frac{\mathrm{d}^4p}{(2\pi)^4}\\
&\times\int_0^\infty \mathrm{d}e_0\exp\left[\mathrm{i}p\cdot(x_b-x_a)+\frac{\mathrm{i}e_0}{2}(p^2-m^2)\right]\\
&\times\int \mathrm{d}\chi_0\chi_0^{1-\lambda}\int \mathrm{d}\varphi_a\mathrm{d}\varphi_b\delta(\varphi_a-k\cdot x_a)\\
&\times\int D\varphi Dp_\varphi\mathcal{D}\Psi\exp\left\{\mathrm{i}\int_0^1\left[e_0\left(gA\cdot p+\frac{g^2}{2}A^2\right)\right.\right.\\
&+p_\varphi(\dot{\varphi}+e_0p\cdot k-\mathrm{i}\lambda k\cdot\Psi\chi_0)\\
&+\mathrm{i}e_0(g+2m\mu)F_{\mu\nu}\Psi^\mu\Psi^\nu\\
&+4\mathrm{i}\mu e_0\Psi^5F_{\mu\nu}[p+gA]^\mu\Psi^\nu\\
&-\mathrm{i}\lambda\left[(p+gA)\cdot\Psi+(m-2\mathrm{i}\mu F_{\mu\nu}\Psi^\mu\Psi^\nu)\Psi^5\right]\chi_0\\
&\left.\left.-\mathrm{i}\Psi_n\dot{\Psi}^n\right]\mathrm{d}\tau+\Psi_n(1)\Psi^n(0)\right\}\Bigg|_{\theta=0}.\end{aligned}\tag{12}$$

It is remarkable to see that the introduction of this constraint has simplified the calculations by bringing the study of the motion from quadridimensional space x^μ to a unidimensional space described by the variable φ. Furthermore, we point out that the coupling term of the spin variables with the electromagnetic field is written as

$$F_{\mu\nu}\Psi^\mu\Psi^\nu = 2(k\cdot\Psi)(A'\cdot\Psi), \tag{13}$$

where the prime indicates a derivative with respect to the argument φ. We then proceed in the same way for the Grassmann case, by suggesting the introduction of a second variable η which considers $k\cdot\Psi$ independent of Ψ via the following identity [10,11]:

$$\int \mathrm{d}\eta_a\mathrm{d}\eta_b\delta(\eta_a-k\cdot\Psi_a)\times\int D\eta Dp_\eta\exp\left[\mathrm{i}\int_0^1 p_\eta\left(\dot{\eta}-k\cdot\dot{\Psi}\right)\mathrm{d}\tau\right]=1, \tag{14}$$

where the variables η and p_η are of the same nature as Ψ, i.e. they are odd (Grassmann) variables.

Knowing that the propagators of the spin variables are subject to the boundary condition $\Psi^n(1)+\Psi^n(0)=\theta^n$, reflecting the antiperiodic character of the spin, and that the exponential contains an additional term $\Psi_n(1)\Psi^n(0)$, it is convenient to elude these complications by passing to the velocity space following the variable change $\Psi(\tau)\to\omega(\tau)$: $\Psi^n(\tau)=(1/2)\int_0^1\varepsilon(\tau-\tau')\omega^n(\tau')\mathrm{d}\tau'+(\theta^n/2)$; $\varepsilon(\tau)$ is the sign of τ, where the velocity $\omega(\tau)$ is an odd (Grassmann) variable.

We should note that during the initial and final time, the so-called antiperiodic boundary condition is always satisfied. In other words, the velocity variables are not subject to any restriction, in contrast to the Ψ^μ. Moreover, following this transformation, a quadratic term in $\omega(\tau)$ has appeared in the action. Therefore, the Green function becomes

$$\begin{aligned}\widetilde{S}_\lambda = &\left(\frac{-\mathrm{i}}{2}\right)^{1-\lambda}\exp\left(\mathrm{i}\widetilde{\gamma}^n\frac{\partial_1}{\partial\theta^n}\right)\int\frac{\mathrm{d}^4p}{(2\pi)^4}\int_0^\infty\mathrm{d}e_0\\
&\times\exp\left[\mathrm{i}p\cdot(x_b-x_a)+\frac{\mathrm{i}e_0}{2}(p^2-m^2)\right]\int\mathrm{d}\chi_0\chi_0^{1-\lambda}\\
&\times\int\mathrm{d}\varphi_a\mathrm{d}\varphi_b\mathrm{d}\eta_a\mathrm{d}\eta_b\delta(\varphi_a-k\cdot x_a)\,\delta\left(\eta_a+\frac{k}{2}\cdot(\omega-\theta)\right)\\
&\times\int D\varphi Dp_\varphi D\eta Dp_\eta\mathcal{D}\omega\\
&\times\exp\left\{\mathrm{i}\int_0^1\left[e_0\left(gA\cdot p+\frac{g^2}{2}A^2\right)\right.\right.\\
&+p_\varphi(\dot{\varphi}+e_0p\cdot k-\mathrm{i}\lambda\eta\chi_0)+p_\eta\left(\dot{\eta}-k\cdot\omega\right)\\
&-\frac{\mathrm{i}\lambda}{2}\left[(p+gA)\cdot(\varepsilon\omega+\theta)+m(\varepsilon\omega^5+\theta^5)\right]\chi_0\\
&+\mathrm{i}e_0(g+2m\mu)\eta A'\cdot(\varepsilon\omega+\theta)\\
&+2\mathrm{i}\mu e_0(\varepsilon\omega^5+\theta^5)\\
&\times\left[\frac{1}{2}\left(p\cdot k-\frac{\mathrm{i}\lambda}{e_0}\eta\chi_0\right)A'\cdot(\varepsilon\omega+\theta)\right.\\
&\left.\left.\left.-\eta A'\cdot(p+gA)\right]+\frac{\mathrm{i}}{2}\omega_n\varepsilon\omega^n\right]\mathrm{d}\tau\right\}\Bigg|_{\theta=0},\end{aligned}\tag{15}$$

where we have used the following notations:

$$A\varepsilon B=\int_0^1 A(\tau)\,\varepsilon(\tau-\tau')B(\tau')\,\mathrm{d}\tau',$$
$$\mathcal{D}\omega=\left(\sqrt{\mathrm{Det}\varepsilon}\right)^{-1}D\omega.$$

In order to extract the classical equation of motion let us make the following shift:

$$\omega^\mu(\tau) \longrightarrow \omega^\mu(\tau) + \mathrm{i}k^\mu \int_0^1 \varepsilon^{-1}(\tau-\tau')p_\eta(\tau')\mathrm{d}\tau'. \qquad (16)$$

Consequently, the integration over $p_\eta(\tau)$ will be straightforward and the Dirac functional appears as

$$\delta(\dot{\eta} + (\lambda p\cdot k/2)\chi_0)$$

which expresses that the contribution to the propagator calculus comes essentially from the path verifying the equation

$$\eta(\tau) = \eta_\lambda(\tau) = \eta_a - \frac{\lambda p\cdot k}{2}\chi_0\tau, \qquad (17)$$

which is the classical equation of motion.

In effect, starting from the Lagrangian present in (9), we derive the classical equations of motion. Then multiplying them by k^μ, we easily get

$$\begin{aligned} p\cdot k &= \text{const}, \\ -\frac{(k\cdot \dot{x})}{e_0} + \frac{\mathrm{i}\lambda(k\cdot \Psi)\chi_0}{e_0} &= p\cdot k, \\ k\cdot\dot{\Psi} - \frac{\lambda\,(k\cdot \dot{x})}{2e_0}\chi_0 &= 0. \end{aligned} \qquad (18)$$

Combining these three equations we obtain (17).

In order to integrate over $\omega(\tau)$, let us go back to integral form of the Dirac function $\delta(\eta_a + (k/2)\cdot(\omega-\theta))$. So, the propagator function related to the $\omega(\tau)$ variables will take the following form:

$$\int \mathcal{D}\omega \exp\left\{\frac{-1}{2}\int_0^1\int_0^1 \omega^n(\tau)\mathcal{M}_{nm}(\tau,\tau')\omega^m(\tau')\mathrm{d}\tau\mathrm{d}\tau' + \int_0^1 J_n(\tau)\omega^n(\tau)\mathrm{d}\tau\right\}\Bigg|_{\theta=0}, \qquad (19)$$

where we have defined

$$\mathcal{M}_{nm} = \begin{bmatrix} \eta_{\mu\nu}\varepsilon & 0 \\ C_{5\nu} & -\varepsilon \end{bmatrix},$$

$$C_{5\nu}(\tau,\tau') = 2\mu e_0\left(p\cdot k - \frac{\mathrm{i}\lambda}{e_0}\eta_a\chi_0\right) \times \int_0^1 A'_\nu[\varphi(\tau_1)]\,\varepsilon(\tau_1-\tau)\varepsilon(\tau_1-\tau')d\tau_1, \qquad (20)$$

$$\begin{aligned} J_\mu(\tau) = &-e_0(g+2m\mu) \times \int_0^1 \eta_\lambda(\tau')\,A'_\mu[\varphi(\tau')]\,\varepsilon(\tau'-\tau)\,\mathrm{d}\tau' \\ &- \frac{\lambda}{2}\chi_0\int_0^1 [p_\mu + gA_\mu[\varphi(\tau')]]\varepsilon(\tau'-\tau)\,\mathrm{d}\tau' \\ &- \mu e_0\left(p\cdot k - \frac{\mathrm{i}\lambda}{e_0}\eta_a\chi_0\right)\theta^5 \\ &\times \int_0^1 A'_\mu[\varphi(\tau')]\,\varepsilon(\tau'-\tau)\,\mathrm{d}\tau' + \frac{\mathrm{i}}{2}k_\mu p_{\eta_a}, \end{aligned} \qquad (21)$$

$$\begin{aligned} J_5(\tau) = &-\frac{m\lambda}{2}\chi_0\int_0^1 \varepsilon(\tau'-\tau)\mathrm{d}\tau' \\ &+ \mu e_0\left(p\cdot k - \frac{\mathrm{i}\lambda}{e_0}\eta_a\chi_0\right) \times \int_0^1 A'[\varphi(\tau')]\cdot\theta\varepsilon(\tau'-\tau)\mathrm{d}\tau' \\ &- 2\mu e_0\int_0^1 \eta_\lambda(\tau')\,A'[\varphi(\tau')]\cdot[p + gA[\varphi(\tau')]] \times \varepsilon(\tau'-\tau)\,\mathrm{d}\tau'. \end{aligned} \qquad (22)$$

The integral over the variable $\omega(\tau)$ is Gaussian and the result is obtained straightforwardly. It is equal to

$$\sqrt{\det\mathcal{M}}\exp\left\{\frac{-1}{2}\int_0^1\int_0^1 J^n(\tau)\mathcal{M}^{-1}_{nm}(\tau,\tau')J^m(\tau')\mathrm{d}\tau\mathrm{d}\tau'\right\}. \qquad (23)$$

The inverse matrix $\mathcal{M}^{-1}$ can be easily calculated using the iteration method and its result is then given by

$$\mathcal{M}^{-1}_{nm} = \begin{bmatrix} \eta_{\mu\nu}\varepsilon^{-1} & 0 \\ C^{-1}_{5\nu} & -\varepsilon^{-1} \end{bmatrix},$$

$$C^{-1}_{5\nu}(\tau,\tau') = -2\mu e_0\left(p\cdot k - \frac{\mathrm{i}\lambda}{e_0}\eta_a\chi_0\right) \times \delta(\tau-\tau')\,A'_\nu[\varphi(\tau)]. \qquad (24)$$

It is easy to verify that $\mathrm{Det}\mathcal{M} = \mathrm{Det}(\varepsilon)$. Accordingly, the Green function is rewritten

$$\begin{aligned} \widetilde{S}_\lambda = &\left(\frac{-\mathrm{i}}{2}\right)^{1-\lambda}\exp\left(\mathrm{i}\widetilde{\gamma}^n\frac{\partial_\mathrm{l}}{\partial\theta^n}\right)\int\frac{\mathrm{d}^4p}{(2\pi)^4}\int_0^\infty \mathrm{d}e_0 \\ &\times\exp\left[\mathrm{i}p\cdot(x_b-x_a) + \frac{\mathrm{i}e_0}{2}(p^2-m^2)\right] \\ &\times\int\mathrm{d}\chi_0\chi_0^{1-\lambda}\int\mathrm{d}\varphi_a\mathrm{d}\varphi_b\mathrm{d}\eta_a\mathrm{d}\eta_b\delta(\varphi_a - k\cdot x_a) \\ &\times\delta\left(\eta_b - \eta_a + \frac{\lambda p\cdot k}{2}\chi_0\right)\int\mathrm{d}p_{\eta_a}\int D\varphi Dp_\varphi \\ &\times\exp\left\{\mathrm{i}\int_0^1\left[p_\varphi(\dot{\varphi} + e_0p\cdot k - \mathrm{i}\lambda\eta_a\chi_0) + p_{\eta_a}\left(\eta_a - \frac{k\cdot\theta}{2} - \frac{\lambda p\cdot k}{4}\chi_0\right) + \sum_{k=0}^{3}\mu^k\mathcal{F}_k(e_0,\chi_0,\theta,\varphi)\,\mathrm{d}\tau\right]\right\}_{\theta=0}, \end{aligned} \qquad (25)$$

where

$$\begin{aligned} \mathcal{F}_0(e_0,\chi_0,\theta,\varphi) = &\,e_0\left(gA\cdot p + \frac{g^2}{2}A^2\right) + \mathrm{i}ge_0\eta_\lambda A'\cdot\theta \\ &-\frac{\mathrm{i}\lambda}{2}\left[(p+gA)\cdot\theta + m\theta^5)\right]\chi_0 \\ &-\frac{\mathrm{i}\lambda ge_0}{2}\eta_a\chi_0\left[e_0p\cdot kg\tau A'\cdot\varepsilon A' + A'\cdot\varepsilon\,(p+gA)\right], \end{aligned}$$

$$\mathcal{F}_1(e_0,\chi_0,\theta,\varphi) = \mathrm{i}e_0\left\{\frac{\lambda}{2}\left(p\cdot k\theta^5 + 2m\eta_a\right)\right.$$

$$\times\left[-e_0pkg\tau A'\cdot\varepsilon A'-A'\cdot\varepsilon\left(p+gA\right)\right]\chi_0$$
$$+2m\eta_\lambda A'\cdot\theta+\left[2\theta^5+m\lambda\chi_0\left(2\tau-1\right)\right]$$
$$\times\left[\frac{1}{2}\left(p\cdot k-\frac{\mathrm{i}\lambda}{e_0}\eta_a\chi_0\right)A'\cdot\theta-\eta_\lambda A'\cdot\left(p+gA\right)\right]\Bigg\},$$

$$\mathcal{F}_2\left(e_0,\chi_0,\theta,\varphi\right)=\mathrm{i}e_0^2\Bigg\{\frac{1}{2}(p\cdot k-\frac{\mathrm{i}\lambda}{e_0}\eta_a\chi_0)^2A'\cdot\theta\varepsilon A'\cdot\theta$$
$$+2\lambda p\cdot k\eta_a\chi_0\tau A'\cdot\left(p+gA\right)\varepsilon A'\cdot\left(p+gA\right)$$
$$-2p\cdot k\eta_\lambda A'\cdot\left(p+gA\right)\varepsilon A'\cdot\theta$$
$$-\frac{\lambda\left(p\cdot k\right)^2}{2}\chi_0A'\cdot\left[\varepsilon\left(p+gA\right)\right]\left(\varepsilon A'\cdot\theta\right)$$
$$+\lambda p\cdot k\chi_0\eta_aA'\cdot\left[\varepsilon\left(p+gA\right)\right]\left[\varepsilon A'\cdot\left(p+gA\right)\right]$$
$$-ge_0\left(p\cdot k\right)^2A'\cdot\left(\varepsilon\eta_\lambda A'\right)\left(\varepsilon A'\cdot\theta\right)$$
$$+2ge_0p\cdot kA'\cdot\left[\varepsilon\eta_\lambda A'\right]\left[\varepsilon\eta_\lambda A'\cdot\left(p+gA\right)\right]\Bigg\},$$

$$\mathcal{F}_3\left(e_0,\chi_0,\theta,\varphi\right)=\mathrm{i}e_0^3\Bigg\{-A'\cdot\Bigg[\varepsilon\Bigg(2m\left(p\cdot k\right)^2\eta_\lambda$$
$$+\left(p\cdot k-\frac{\mathrm{i}\lambda}{e_0}\eta_a\chi_0\right)^3\theta^5\Bigg)A'\Bigg]\left(\varepsilon A'\cdot\theta\right)$$
$$+2p\cdot kA'\cdot\left[\varepsilon\left(p\cdot k\theta^5+2m\eta_\lambda\right)A'\right]$$
$$\times\left[\varepsilon\eta_\lambda A'\cdot\left(p+gA\right)\right]\Bigg\}.$$

The integration over p_{η_a} gives

$$\delta(\eta_a-(k\cdot\theta/2)-(\lambda p\cdot k/4)\chi_0)$$

which fixes

$$\eta_a=\frac{k\cdot\theta}{2}+\frac{\lambda p\cdot k}{4}\chi_0,\qquad(26)$$

assuring that the boundary condition is always satisfied: $\eta_b+\eta_a=k\cdot\theta\mid_{\theta=0}$.

The integration over p_φ gives

$$\delta(\dot{\varphi}+e_0p\cdot k-(\mathrm{i}\lambda k\cdot\theta/2)\chi_0)\ ,$$

which imposes on the path $\varphi\left(\tau\right)$ the function of verifying the classical equation of motion

$$\varphi\left(\tau\right)=\varphi_a-\left(e_0p\cdot k-\frac{\mathrm{i}\lambda k\cdot\theta}{2}\chi_0\right)\tau.\qquad(27)$$

This latter equation concords with the second equations of (18) obtained from the classical Lagrangian Let us also note that for the global projection ($\lambda=0$), the spin does not intervene on the classical exterior motion. In order to integrate over φ_b and φ_a, we insert the integral form of the Dirac function $\delta(\varphi_b-\varphi_a+e_0p\cdot k-(\mathrm{i}\lambda k\cdot\theta/2)\chi_0)$ and make the following shift: $p_\mu\to p_\mu-k_\mu p_{\varphi_b}$. Thus, integrating over χ_0 and e_0, and changing $p_\mu\to-p_\mu$, we obtain for the Green function the following expression:

$$\begin{pmatrix}\widetilde{S}_{\mathrm{l}}\\ G_{\mathrm{g}}\end{pmatrix}=\frac{1}{(2\pi)^4}\int\frac{\mathrm{d}^4p}{(p^2-m^2+\mathrm{i}0)}\begin{pmatrix}\widetilde{\Phi}_{\mathrm{l}}(p,x_b,x_a)\\ \widetilde{\Phi}_{\mathrm{g}}(p,x_b,x_a)\end{pmatrix}\qquad(28)$$
$$\times\exp\left[-\mathrm{i}p\cdot\left(x_b-x_a\right)-\frac{\mathrm{i}g}{p\cdot k}\int_{k\cdot x_a}^{k\cdot x_b}\left(A\cdot p-\frac{g}{2}A^2\right)\mathrm{d}\varphi\right],$$

where the spin factors have the following forms:

$$\begin{pmatrix}\widetilde{\Phi}_{\mathrm{l}}(p,x_b,x_a)\\ \widetilde{\Phi}_{\mathrm{g}}(p,x_b,x_a)\end{pmatrix}=\exp\left(\mathrm{i}\widetilde{\gamma}^n\frac{\partial_{\mathrm{l}}}{\partial\theta^n}\right)\begin{pmatrix}\mathrm{i}\mathcal{L}_n\theta^n\\ 1\end{pmatrix}$$
$$\times\exp\left(\boldsymbol{\theta}^\mu\mathcal{R}_{\mu\nu}\theta^\nu+\theta^5\mathcal{R}_{5\mu}\theta^\mu\right)\Bigg|_{\theta=0},\qquad(29)$$

with

$$\mathcal{R}^{\mu\nu}=\frac{-\left(g+2m\mu\right)}{2p\cdot k}k^\mu\left(A_b-A_a\right)^\nu$$
$$+\mu^2\left(A_b^\mu A_a^\nu+k^\mu\left(A_b+A_a\right)^\nu G^-\right)$$
$$+\int_{k\cdot x_a}^{k\cdot x_b}\mathrm{d}\varphi\Bigg\{\mu^2\Big(-A'^\mu A^\nu$$
$$+\frac{1}{p\cdot k}k^\mu A^\nu A'\cdot\left(2p-g\left(A_b+A_a\right)\right)\Big)$$
$$+2\mu^3mk^\mu S'A^\nu\Bigg\},$$

$$\mathcal{R}_{5\mu}=\mu\left(\left(A_b-A_a\right)_\mu+k_\mu G^-\right)$$
$$-2p\cdot k\mu^3\int_{k\cdot x_a}^{k\cdot x_b}S'\left(A_\mu+k_\mu G\right)\mathrm{d}\varphi,$$

$$\mathcal{L}_5=m+\frac{\mu g}{2}\left(A_b-A_a\right)^2+4p\cdot k\mu^3\Bigg(A_b\cdot A_aG_b$$
$$+\int_{k\cdot x_a}^{k\cdot x_b}\mathrm{d}\varphi\left(p\cdot kG'S-\frac{1}{k\cdot\left(x_b-x_a\right)}A_b\cdot A_aG\right)$$
$$-\frac{p\cdot k}{k\cdot\left(x_b-x_a\right)}\int_{k\cdot x_a}^{k\cdot x_b}\mathrm{d}\varphi\int_{k\cdot x_a}^{\varphi}\mathrm{d}\varphi'G'S\Bigg),$$

$$\mathcal{L}_\mu=-p_\mu+\frac{g}{2}\left(A_b+A_a\right)_\mu$$
$$+\frac{g}{2p\cdot k}k_\mu\left[gA_b\cdot A_a-p\cdot\left(A_b+A_a\right)\right]$$
$$-\frac{\mu gm}{2p\cdot k}k_\mu\left(A_b-A_a\right)^2$$
$$+\mu^2\Big[-\frac{g}{2}\left(A_b+A_a\right)_\mu\left(A_b-A_a\right)^2$$
$$-k_\mu\left(G^+\left(gA_a^2+p\cdot\left(A_b-A_a\right)\right)-gA_b\cdot A_aG^-\right)\Big]$$
$$+2\mu^3m\left(\left(A_b+A_a\right)_\mu+k_\mu G^-\right)A_b\cdot A_a$$
$$+\int_{k\cdot x_a}^{k\cdot x_b}\mathrm{d}\varphi\Bigg\{\mu^2\Bigg(gA'\cdot\left(A_b-A_a\right)A_\mu$$
$$+2k_\mu\left[A'\cdot\left(p-gA_a\right)+\frac{1}{k\cdot\left(x_b-x_a\right)}A_b\cdot A_a\right]G\Bigg)$$
$$+2\mu^3m\Bigg\{-2A'\cdot\Big(2A-A_b$$

$$
\begin{aligned}
&- \frac{1}{k\cdot(x_b-x_a)}\varepsilon(\varphi)A\Big)A_\mu - \frac{1}{k\cdot(x_b-x_a)}\\
&\times\Big((A_b+A_a)_\mu + k_\mu G^+\Big)\left(A^2 + A'\varepsilon(\varphi)A\right)\\
&- 2k_\mu\Bigg[A'\cdot A_a - \frac{1}{k\cdot(x_b-x_a)}\\
&\times(A_b\cdot A_a + A'\cdot\varepsilon(\varphi)A)\Bigg]G\Bigg\}\Bigg\}\\
&+ \frac{2}{k\cdot(x_b-x_a)}\int_{k\cdot x_a}^{k\cdot x_b}\mathrm{d}\varphi\int_{k\cdot x_a}^{\varphi}\mathrm{d}\varphi'\Bigg\{\mu^2 p\cdot k k_\mu G'S\\
&+ 4\mu^3 m\left(A'\cdot A\,(A_\mu + k_\mu G) + \frac{p\cdot k}{2}G'S\right)\Bigg\},
\end{aligned}
$$

where the prime indicates a derivative with respect to the argument φ.

We also notice that we have used the following notations in order to write the previous expressions in a convenient form:

$$
\begin{aligned}
G &= \frac{-1}{p\cdot k}\left(A\cdot p - \frac{g}{2}A^2\right),\\
G^{\pm} &= G_b \pm G_a,\\
S &= \frac{1}{p\cdot k}A\cdot[A-(A_b+A_a)],\\
\varepsilon(\varphi)A &= \int_{kx_a}^{\varphi}A(\varphi')\,\mathrm{d}\varphi' - \int_{\varphi}^{kx_b}A(\varphi')\,\mathrm{d}\varphi'.
\end{aligned}
$$

Finally, to extract explicitly the spin factor, let us proceed to the derivation over the θ variables. The calculations are done in the usual way [9–11], by acting the operator $\partial_l/\partial\theta^n$ and then replacing the θ variables by the $\widetilde{\gamma}^n$ matrices. In both the case of the global and the local derivation, it is very easy to obtain the following results for the spin factor:

$$
\begin{aligned}
&\widetilde{\Phi}_{\mathrm{l}}(p,x_b,x_a)\\
&= -\gamma^5\mathcal{L}_\mu\Big[\gamma^\mu\left(1+\mathrm{i}\mathcal{R}_{\rho\delta}\sigma^{\rho\delta}\right) - \mathrm{i}\mathcal{R}_{5\nu}\left(\sigma^{\mu\nu} - 2\mathrm{i}\mathcal{R}^{*\mu\nu}\gamma^5\right)\\
&+ \left(\mathcal{R}^\mu_{\ \nu} - \mathcal{R}_\nu^{\ \mu}\right)\gamma^\nu\Big] - \gamma^5\mathcal{L}_5\left[1+\mathrm{i}\mathcal{R}_{\mu\nu}\sigma^{\mu\nu} + \mathcal{R}_{\mu\nu}\mathcal{R}^{*\mu\nu}\gamma^5\right],\\
&\widetilde{\Phi}_{\mathrm{g}}(p,x_b,x_a)\\
&= \left(1+\mathcal{R}_{5\rho}\gamma^\rho\right)\left(1+\mathrm{i}\mathcal{R}_{\mu\nu}\sigma^{\mu\nu}\right)\\
&+ \mathcal{R}_5^\mu\left[\mathcal{R}_{\mu\nu} - \mathcal{R}_{\nu\mu}\right]\gamma^\nu + \mathcal{R}_{\mu\nu}\mathcal{R}^{*\mu\nu}\gamma^5, \qquad (30)
\end{aligned}
$$

with $\mathcal{R}^{*\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\delta}R_{\rho\delta}$ ($\epsilon^{\mu\nu\rho\delta}$ is the Lévi-Civita tensor). Using $S^{\mathrm{c}} = -\widetilde{S}\gamma^5$ and according to (7), the dynamics of the system is thus totally determined by the following expression in the local and global representations:

$$
\begin{aligned}
S^{\mathrm{c}} &= -\int\frac{\mathrm{d}^4 p}{(2\pi)^4(p^2-m^2+\mathrm{i}0)}\Phi_p(x_b,x_a) \qquad (31)\\
&\times\exp\left[-\mathrm{i}p\cdot(x_b-x_a) - \frac{\mathrm{i}g}{p\cdot k}\int_{k\cdot x_a}^{k\cdot x_b}\left(A\cdot p - \frac{g}{2}A^2\right)\mathrm{d}\varphi\right],
\end{aligned}
$$

where

$$
\Phi_p(x_b,x_a) = \Phi_p^{\mathrm{l}}(x_b,x_a) = \Phi_p^{\mathrm{g}}(x_b,x_a), \qquad (32)
$$

with

$$
\begin{aligned}
\Phi_p^{\mathrm{l}}(x_b,x_a) &= \mathcal{L}_\mu\Big[\gamma^\mu\left(-1+\mathrm{i}\mathcal{R}_{\rho\delta}\sigma^{\rho\delta}\right)\\
&- \mathrm{i}\mathcal{R}_{5\nu}\left(\sigma^{\mu\nu} - 2\mathrm{i}\mathcal{R}^{*\mu\nu}\gamma^5\right) - \left(\mathcal{R}^\mu_{\ \nu} - \mathcal{R}_\nu^{\ \mu}\right)\gamma^\nu\Big]\\
&+ \mathcal{L}_5\left[1+\mathrm{i}\mathcal{R}_{\mu\nu}\sigma^{\mu\nu} + \mathcal{R}_{\mu\nu}\mathcal{R}^{*\mu\nu}\gamma^5\right], \qquad (33)
\end{aligned}
$$

and

$$
\begin{aligned}
\Phi_p^{\mathrm{g}}(x_b,x_a) &= \Bigg[\widehat{p} + m - g\widehat{A}_b + \mathrm{i}\mu\widehat{k}\widehat{A'}_b\\
&+ \frac{1}{p\cdot k}\widehat{k}\left(gA_b\cdot p - \frac{g^2}{2}A_b^2\right) + \mathrm{i}\widehat{k}\left(\frac{\partial}{\partial(k\cdot x_b)}\right)\Bigg]\\
&\times\Big[\left(1-\mathcal{R}_{5\rho}\gamma^\rho\right)\left(1+\mathrm{i}\mathcal{R}_{\mu\nu}\sigma^{\mu\nu}\right)\\
&- \mathcal{R}_5^\mu\left[\mathcal{R}_{\mu\nu} - \mathcal{R}_{\nu\mu}\right]\gamma^\nu + \mathcal{R}_{\mu\nu}\mathcal{R}^{*\mu\nu}\gamma^5\Big], \qquad (34)
\end{aligned}
$$

and where we have used the notation $\widehat{a} = a\cdot\gamma$.

The propagator related to our problem, (31), has thus been calculated exactly and analytically. It is easy to find out that this propagator is a solution of (1).

This is our main result.

To extract the wave functions from the analytical expression of the propagator (31), it is necessary to perform the symmetrization of the latter. The calculus becomes very long but in principle it causes no difficulty.

Then, to be more effective, it would be interesting to get the exact and explicit result in particular cases such as

(1) a charged particle with a weak anomalous magnetic moment, and
(2) a neutral Dirac particle with a weak anomalous magnetic moment.

3 Some special cases

3.1 Charged particle with a weak anomalous magnetic moment

For this special case we will consider just the first order in μ. The Polyakov spin factor will be given in both the cases of global and local projections by

$$
\begin{aligned}
\Phi_p(x_b,x_a) &= (\widehat{p}+m)\left[1+\frac{g}{2pk}\widehat{k}(\widehat{A}_b - \widehat{A}_a)\right] - g\widehat{A}_b\\
&+ \frac{g}{p\cdot k}\widehat{k}A_b\cdot p - \frac{g^2}{2p\cdot k}\widehat{k}\widehat{A}_b\widehat{A}_a\\
&+ \mu\Bigg[(\widehat{p}+m-g\widehat{A}_b)\\
&\times\left(\frac{m}{p\cdot k}\widehat{k}(\widehat{A}_b-\widehat{A}_a) - (\widehat{A}_b-\widehat{A}_a) - \widehat{k}G^-\right)\\
&+ g\widehat{k}(\widehat{A}_b-\widehat{A}_a)G_b\Bigg]. \qquad (35)
\end{aligned}
$$

Then, after the symmetrization, we easily get the following result:

$$
\begin{aligned}
S^{\mathrm{c}}(x_b,x_a) = & -\frac{1}{(2\pi)^4}\int \frac{\mathrm{d}^4p}{(p^2-m^2+\mathrm{i}0)} \\
& \times \exp\left\{\frac{g}{2p\cdot k}\widehat{k}\widehat{A}_b \right.\\
& \left.+\ \mu\left[\frac{m}{p\cdot k}\widehat{k}\widehat{A}_b+\widehat{A}_b-\frac{1}{p\cdot k}\widehat{k}A_b\cdot p\right]\right\} \\
& \times(\widehat{p}+m)\exp\left\{\frac{-g}{2p\cdot k}\widehat{k}\widehat{A}_a \right.\\
& \left.+\ \mu\left[\frac{-m}{p\cdot k}\widehat{k}\widehat{A}_a+\widehat{A}_a-\frac{1}{p\cdot k}\widehat{k}A_a\cdot p\right]\right\} \\
& \times \exp\left\{-\mathrm{i}p\cdot(x_b-x_a)\right.\\
& \left.-\frac{\mathrm{i}g}{p\cdot k}\int_{k\cdot x_a}^{k\cdot x_b}\left(A\cdot p-\frac{g}{2}A^2\right)\mathrm{d}\varphi\right\}.
\end{aligned} \tag{36}
$$

In order to determine the wave functions, let us integrate over the energy p^0 and carry out the projection on the positive and negative energy states [19]:

$$
\Lambda_+(p)=\sum_{\pm s}u(p,s)\,\overline{u}(p,s)=\frac{\widehat{p}+m}{2m}, \tag{37}
$$

$$
\Lambda_-(p)=-\sum_{\pm s}v(p,s)\,\overline{v}(p,s)=\frac{-\widehat{p}+m}{2m};
$$

we thus obtain the following form for $S^{\mathrm{c}}(x_b,x_a)$:

$$
\begin{aligned}
S^{\mathrm{c}}(x_b,x_a) = & +\mathrm{i}\theta(t_b-t_a)\int \mathrm{d}^3p\sum_{\pm s}\psi^{(+)}_{s,\boldsymbol{p}}(x_b)\,\overline{\psi}^{(+)}_{s,\boldsymbol{p}}(x_a) \\
& -\mathrm{i}\theta(t_a-t_b)\int \mathrm{d}^3p\sum_{\pm s}\psi^{(-)}_{s,\boldsymbol{p}}(x_b)\,\overline{\psi}^{(-)}_{s,\boldsymbol{p}}(x_a),
\end{aligned} \tag{38}
$$

where the normalized wave functions describing the motion of the Dirac particles are

$$
\begin{aligned}
\psi^{(+)}_{s,\boldsymbol{p}}(x) = & \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{p^0}\right)^{1/2} \\
& \times\exp\left\{\frac{g}{2p\cdot k}\widehat{k}\widehat{A}+\mu\left[\frac{m}{p\cdot k}\widehat{k}\widehat{A}+\widehat{A}-\frac{1}{p\cdot k}\widehat{k}A\cdot p\right]\right\} \\
& \times u(p,s) \\
& \times\exp\left\{-\mathrm{i}p\cdot x-\frac{\mathrm{i}g}{p\cdot k}\int_{k\cdot x_0}^{k\cdot x}\left(A\cdot p-\frac{g}{2}A^2\right)\mathrm{d}\varphi\right\},
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
\psi^{(-)}_{s,\boldsymbol{p}}(x) = & \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{p^0}\right)^{1/2} \\
& \times\exp\left\{\frac{-g}{2p\cdot k}\widehat{k}\widehat{A}+\mu\left[\frac{-m}{p\cdot k}\widehat{k}\widehat{A}+\widehat{A}-\frac{1}{p\cdot k}\widehat{k}A\cdot p\right]\right\} \\
& \times v(p,s) \\
& \times\exp\left\{\mathrm{i}p\cdot x-\frac{\mathrm{i}g}{p\cdot k}\int_{k\cdot x_0}^{k\cdot x}\left(A\cdot p+\frac{g}{2}A^2\right)\mathrm{d}\varphi\right\},
\end{aligned} \tag{40}
$$

where $p^0=\left(\boldsymbol{p}^2+m^2\right)^{1/2}$; $k\cdot x_0$ is a constant and $u(p,s)$ and $v(p,s)$ are the spinors which are the solutions of the free Dirac equation; $\overline{u}(p,s)\,u(p,s)=1$ and $\overline{v}(p,s)\,v(p,s)=-1$.

In the case of a charged particle without the anomalous magnetic moment, we suppress the anomalous magnetic moment μ in (39) and (40). Thus, the wave functions are

$$
\begin{aligned}
\psi^{(+)}_{s,\boldsymbol{p}}(x) = & \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{p^0}\right)^{1/2}\left[1+\frac{g}{2p\cdot k}\widehat{k}\widehat{A}\right]u(p,s) \\
& \times\exp\left\{-\mathrm{i}p\cdot x-\frac{\mathrm{i}g}{p\cdot k}\int_{k\cdot x_0}^{k\cdot x}\left(A\cdot p-\frac{g}{2}A^2\right)\mathrm{d}\varphi\right\},
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
\psi^{(-)}_{s,\boldsymbol{p}}(x) = & \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{p^0}\right)^{1/2}\left[1-\frac{g}{2p\cdot k}\widehat{k}\widehat{A}\right]v(p,s) \\
& \times\exp\left\{\mathrm{i}p\cdot x-\frac{\mathrm{i}g}{p\cdot k}\int_{k\cdot x_0}^{k\cdot x}\left(A\cdot p+\frac{g}{2}A^2\right)\mathrm{d}\varphi\right\},
\end{aligned} \tag{42}
$$

The result agrees with the literature [9–11]

3.2 Neutral Dirac particle with a weak anomalous magnetic moment

For this special case we will consider the neutral particle with a weak anomalous magnetic moment. We put $g=0$ in (39) and (40); thus, the wave functions are

$$
\begin{aligned}
\psi^{(+)}_{s,\boldsymbol{p}}(x) = & \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{p^0}\right)^{1/2} \\
& \times\exp\left\{\mu\left[\frac{m}{p\cdot k}\widehat{k}\widehat{A}+\widehat{A}-\frac{1}{p\cdot k}\widehat{k}A\cdot p\right]\right\} \\
& \times u(p,s)\exp(-\mathrm{i}p\cdot x),
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
\psi^{(-)}_{s,\boldsymbol{p}}(x) = & \frac{1}{(2\pi)^{3/2}}\left(\frac{m}{p^0}\right)^{1/2} \\
& \times\exp\left\{\mu\left[\frac{-m}{p\cdot k}\widehat{k}\widehat{A}+\widehat{A}-\frac{1}{p\cdot k}\widehat{k}A\cdot p\right]\right\} \\
& \times v(p,s)\exp(\mathrm{i}p\cdot x).
\end{aligned} \tag{44}
$$

This result is in accordance with that of [20].

4 Conclusion

We have shown through this problem of the particle with anomalous magnetic moment that the propagator of this particle is exactly and analytically calculable by two approaches: the global and the local one, and due to two identities, the first one related to x (time-space), and the second one related to Ψ (spin-space).

The two identities we introduced have allowed us to reduce the contributions of all the paths to a calculus of the propagators with principally classical paths projected along the direction of the plane wave propagation k.

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